T. Mase and A. M. Kersten, ”Experimental evaluation of a 3-D hyperelastic, rate-dependent golf ball constitutive model,” The Engineering of Sport 5 - Conference Proceedings of, Vol. 2, pp. 238-244, 2004

A comparison of 3-D hyperelastic, rate-dependent and 1-D nonlinear viscoelastic golf ball constitutive models for simulating impact

Tom Mase
Ann Marie Kersten
Composite Materials and Structures Center
Michigan State University, East Lansing, MI, USA

ABSTRACT: Experimental measurements of golf ball covers and cores have been conducted providing a visco-hyperelastic material model which may be used in finite element impact simulation. The material model parameters found accurately predict coefficient of restitution of a ball obliquely impacting a rigid plate for impact speeds ranging from 35 to 55 m/s.

INTRODUCTION

In striving for larger driver heads with large coefficients of restitutions the simulation and modeling of impact have become more and more important. Not only does simulation help with deciding club face structural fidelity and weld line forces, but it can also be used to evaluate shot initial launch conditions. The modeling of this impact involves somewhat complex constitutive relationships involving large deflections and rate-dependent behavior.

Modern golf balls have changed from wound balata construction to 2-, 3- and 4-piece solid construction (Nesbit 1999). Materials used in ball covers are generally ionomers and the cores are highly crosslined polybutadiene (Statz 1990, 1999). Experimental characterization of the golf ball structure is generally done by impelling the ball against a rigid force plate (Tavares 1999, Cochran 2002). Considerable work has been done using nonlinear viscoelastic models of the ball structure to model the oblique impact (Cochran 2002, Johnson 1999) in addition to finite element simulation.

This paper reports the experimental results obtained by benchmarking several commercially available golf balls to determine material properties for use in impact simulation. Thermoplastic covers were cut off and re-molded into standard dynamic modulus analysis (DMA) specimens using a DSM micro extruder/injection molding machine following published Surlyn recommended molding procedures. Flexure specimens were cut out of the cores for DMA testing. Initial stress-strain response was done at room temperature to get quasi-static response. Flexure relaxation testing was done at various temperatures and time-temperature superposition was used to generate a master curve referred to room temperature. The experimental results were used in a 3-D hyperelastic, rate-dependent material model and compared to previously published 1-D viscoelastic model and experimental data.

COVER AND CORE SPECIMEN PREPARATION

The ball outer covers were removed by making circumferential cuts through the outermost layer using a band saw. A flat head screw driver was inserted into the intersection of the two cuts and twisted to start the removal of the cover. The ProV1 utilizes an outer cast polyurethane cover and an inner Surlyn casing. In this case the outer urethane cover was first removed and the process repeated to remove the Surlyn casing. During all of the ball deconstruction it was observed that the material's temperature never rose to noteworthy levels. Once the covers were removed, the thermoplastic materials (Pinnacle Gold cover and Pro V1 casing) were cut into small pieces (< 1 cm) using diagonal pliers. Small pieces were needed for subsequent molding into test flexure and tension specimens. Molding was done with a mini-injection molding machine (DSM) using process parameters based on DuPont's molding recommendations (Dupont 2004). This process allows for the making of tensile bars from a limited amount of material. A molding pressure of 1.034 MPa was used (nitrogen assisted) as well as a holding cylinder temperature of 240 $^{\circ}C$ and a mold temperature of 41 $^{\circ}C$. Cover material and subsequent test specimens are shown in Figure 1a.

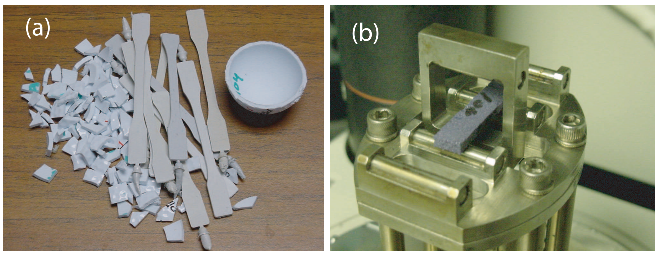

Figure 1: (a) Pinnacle Gold cover, cover material, and tensile test specimens, (b) core flexure specimen in 3-point bend DMA fixture

The Pinnacle Gold and the ProV1 cores were turned down to 31.75 mm cylinders using a high rpm lathe. Next, the end of the ball was face cut flat by chucking the cylinder in a three jaw, self centering chuck. From this cylindrical shape ASTM D575 specimens could be made for compression testing. In addition, the cylindrical shape was used as the starting point for cutting/machining small flexure specimens (20 x 6.25 x 2.65 mm) that would be used for dynamic modulus analysis (DMA) (Fig. 1b).

COVER/CASING TESTING

It is well known that the cover of the Pinnacle Gold ball is Surlyn. To make certain the Pro V1 casing material was Surlyn, thus defining the molding parameters, spectral data was collected on the two materials using FTIR. Thin films were pressed using a Carver single daylight press preheated to 170 $^\circ C$. Thin aluminum sheets were coated with sintered release agent (MONO-COAT 1472W sintered at 335 $^\circ C$) to protect the platens. The FTIR graphs produced similar enough results to declare the casing material an ionomer.

Specimen dimensions for cover flexure testing were 61.25 x 12.5 x 3.175 mm. Room temperature, static modulus properties were measured using a DMA (TA Instruments Model 2980) in strain sweep mode. The span for the 3-point bend test was 50 mm and a maximum strain of 0.02% was used with a frequency of 1 Hz. For these tests, the elastic (storage) modulus was measured as 390 and 360 MPa for the Pinnacle Gold cover and Pro V1 casing, respectively. Tensile specimens (shown in Fig. 1a) tested in a universal electromechanical test machine (UTS SFM-20, 100-lb load cell, laser extensometer EXT-62-LHM0) resulted in lower elastic modulus values of 329 (7% COV) and 276 (5% COV) MPa for Pinnacle Gold cover and Pro V1 casing, respectively. Poisson's ratio was measured in the tensile test as well and found to be 0.410 (15% COV) and for 0.433 (9% COV) for Pinnacle Gold cover and Pro V1 casing, respectively. Poisson's ratio was calculated from strain acquired from clip-on extensometers (Epsilon Technology Corp., models 3542-0100-010-HT2 (longitudinal) and 3575-020-HT2(transverse)). The reason the DMA modulus values were higher than the uniaxial was the strain rate was higher (0.02 s^{-1} versus 0.008 s^{-1}). Uniaxial testing allowed for measuring the ultimate strength and post-yield tangent modulus. Both materials have failure strains above 50%. Table 1 summarizes quasi-static material properties for Pinnacle Gold covers and Pro V1 casing.

Table 1 Cover and casing properties

Material	Modulus (MPa)	Poisson's Ratio	Strength (MPa)	Tangent Modulus (MPa)
Pinnacle Gold Cover	329	0.433	21.9	14.09
Pro V1 casing	276	0.410	12.81	—

CORE TESTING FOR A RATE DEPENDENT, HYPERELASTIC MODEL

To characterize the core as a rate-dependent hyperelastic material two types of tests were completed: modulus and stress relaxation. Both of these tests were easily done using a DMA test machine. The modulus was determined from conducting a strain sweep at a fixed frequency. In this test method, the specimen was flexed at 1 Hz in a prescribed increasing sequence of five strains. Force, and thus stress, were also recorded at each strain level. Modulus was found from a linear fit of the stress-strain pairs.

Even though the DMA testing was done at small strain, it well represents the response of the polybutadiene core. Modestly nonlinear elastic response is well modeled as a Mooney-Rivlin material having two constants A and B from which stress is found by $\sigma = 2\left(\lambda^2 + \lambda^{-1}\right)\left(A + B\lambda^{-1}\right)$. Testing has shown that golf ball core response may be modeled as a neo-Hookean rather than a Mooney-Rivlin material as can be seen in Fig. 2. Here, stress-strain data from an ASTM

D575 specimen loaded in compression shows minimal nonlinear response up to 30% strain. Thus, a single parameter was all that was needed to represent the core response for strains up to 30%. Setting Mooney-Rivlin constant $B = 0$ reduces the model to a neo-Hookean model. Furthermore, since the rubber core is essentially incompressible, the modulus is related to the hyperelastic constant A through

$$G = \frac{E}{2\,(1+\nu)} = 2\,(A+B) \Rightarrow E = 6A \tag{1}$$

where A and B are Mooney-Rivlin constants and $\nu = 0.5$ is Poisson's ratio for an incompressible material.

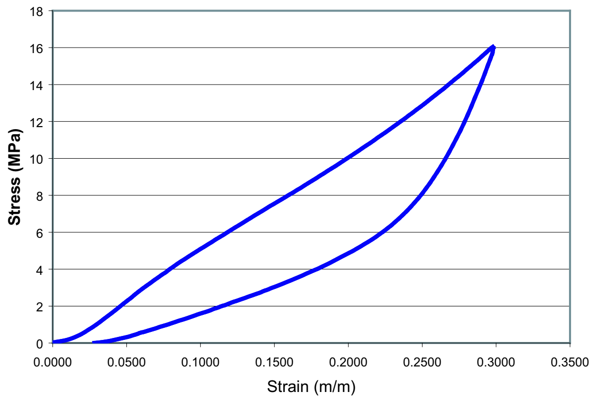

Figure 2: Stress-strain loading and unloading curve for a Pinnacle Gold core ASTM D575 compression test. Specimen was a 28.6 mm right cylinder that is 12.5 mm in height.)

The second test needed to add time-dependent response to the material model is a relaxation test. Once again, this test was done in 3-point flexure in a DMA test machine. Since the golf ball impact occurs in a short period of time, a sequence of relaxation tests were conducted at different temperatures in order to construct a master relaxation curve (Ferry 1961) spanning several decades of time. The master curve was then fit to a Prony series of the form

$$g\,(t) = \sum_{i=1}^{6} G_i e^{-\beta_i t} \tag{2}$$

where $g\,(t)$ is the relaxation function of the material in excess to the hyperelastic static response. In general, the stress rate effect is accounted for by adding to the hyperelastic response stress defined by the convolution integral

$$\sigma_{ij} = \int_0^t g_{ijk\ell}\,(t-\tau)\,\frac{\partial \varepsilon_{k\ell}}{\partial \tau}\,d\tau \tag{3}$$

The Prony series of Eq. 2 is used since simple rate effects were deemed sufficient for modeling the golf ball core (LS-DYNA, 1999).

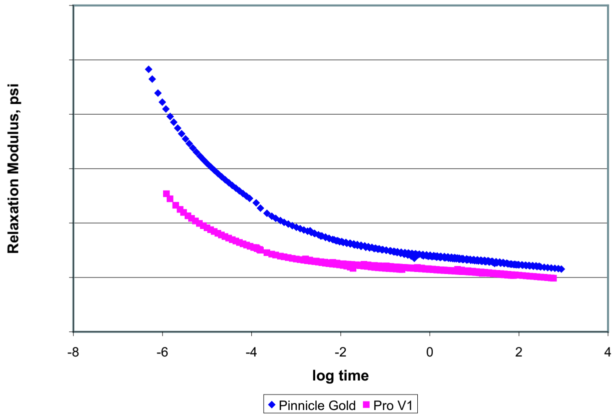

Figure 3: Relaxation master curves reduced to room temperature for two core materials.

A series of flexure relaxation tests were completed at temperatures ranging from -90 $^\circ C$ to room temperature. Relaxation modulus data were shifted onto a master curve referenced at room temperature for determining the Prony series coefficients. The master curves for two core materials are shown in Fig. 3. The master curve data are fit to the Prony series model using Matlab. For all of the data a matrix $\mathbf{A}$ was formed having the columns $e^{-\beta_i \mathbf{t}}$ where $\mathbf{t}$ is a vector of times. The linear coefficients, G_i, were solved for and the difference between this approximation and data was minimized to solve for β_i. This unconstrained minimization was done using a Nelder-Mead simplex method (Nelder 1965). The relaxation fit for the Pinnacle Gold core are given in Table 2. Parameters in this Table are sufficient for modeling visco-hyperelastic behavior in LS-DYNA, a commercially available finite element code.

Table 2 Pinnacle Gold core neo-Hookean material properties ($B \equiv 0$)

Parameter		$i=1$	$i=2$	$i=3$	$i=4$
A, (MPa)	10.00				
G_i, (MPa)		3.075	2.834	11.56	20.56
β_i		70	250	8 130	324 470

IMPACT RESULTS

Having done the experiments to characterize the core and cover material for the Pinnacle Gold ball, impact simulation was carried out to validate the model. In addition to the viscoelastic, hyperelastic material model simulation, a 1-D nonlinear viscoelastic model was also considered (Cochran, 2002). In this work, the impact of an unnamed ball with a rigid plate was modeled as follows:

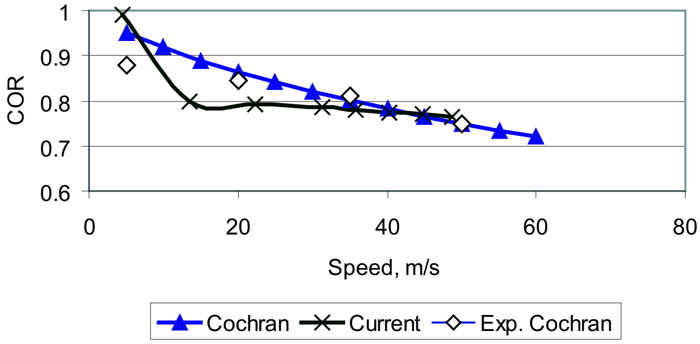

Figure 4: Coefficient of restitution for 1-D model, visco-hyperelastic model, and experimental results.

$$m_{ball}\ddot{x} = -kx\,|x|^a - c\dot{x}\,|x|^b \tag{4}$$

It is a simple task to set up the initial value problem and integrate to get a model of the impact. Having solved for the velocity and position, one can readily calculate the coefficient of restitution (COR) and contact time.

For finite element modeling with the viscoelastic, hyperelastic core material a spherical ball model was made using solid elements. Elements in the outer 2.29 mm of the sphere were given cover properties (elastic-plastic). The remainder of the elements were given core material properties (visco-hyperelastic). This ball was given an initial velocity and impacted with a single, fixed rigid element. Default contact parameters were used. Once again, the coefficient of restitution and time of contact were determined. All finite element simulations were conducted using LS-DYNA.

Figure 4 shows a plot of the coefficient of restitution for a wide range of initial velocities. Plotted in this figure are data from Cochran's model as well as experimental results (Cochran, 2002). Both the 1-D model and visco-hyperelastic models well represent the experiments between 35 and 55 m/s. For impact speeds below 30 m/s the current visco-hyperelastic model underestimates COR. Impact time is another important parameter in validating simulation material properties. Figure 5 shows the 3-D visco-hyperelastic model presented in this paper predicts impact times that are less than the experimental data.

CONCLUSIONS

A method for determining the rate-dependent mechanical response of golf ball thermoplastic covers and cores was developed. The properties for Pinnacle Gold ball were measured. These properties were used in finite element simulation to model the ball's impact with a rigid plate. Simulation coefficient of restitution results show that the visco-hyperelastic model closely matches prior experimental data in the range of velocities between 35 and 55 m/s.

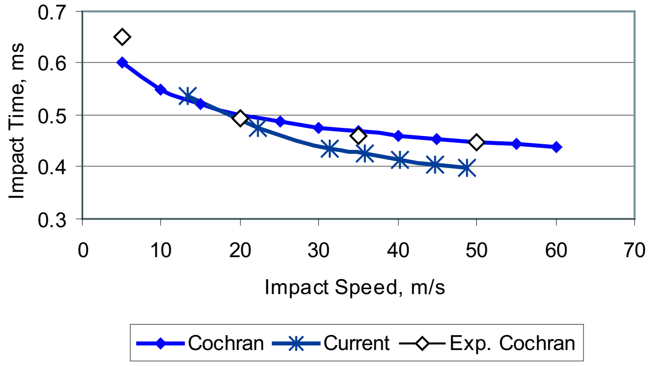

Figure 5: Impact time for 1-D model, visco-hyperelastic model, and experimental results.

REFERENCES

Cochran, Alastair J., 2002, "Development and use of one-dimensional models of a golf ball," *Journal of Sports Sciences*, Taylor & Francis, Ltd. London, UK

Dupont, 2004, web site http://www.dupont.com/industrial-polymers/surlyn/H-37063-2/H-37063-2.html

Ferry, John D., 1961, *Viscoelastic Properties of Polymers*, Wiley & Sons, Inc., New York, NY, pp.203 ff

Johnson, S.H. and J.E. Hubbell, 1999, "Golf Ball Rebound Enhancement," *Science and Golf III: Proceedings of the 1998 World Scientific Congress of Golf*, M.R. Farrally and A.J. Cochran, Editors, Human Kinetics, Champaign, IL, USA, pp. 493-499

LS-DYNA Keyword Users Manual, Version 950, 1999, Livermore Software Technology Corporation, Livermore, CA, p. 20-227

Nelder, J.A. and R. Mead, 1965 "A Simplex Method for Function Minimization," *Computer J.*, Vol. 7, pp. 308-313

Nesbitt, R.D., M.J. Sullivan, and T. Melvin, 1999, "History and Construction of Non-Wound Golf Balls," *Science and Golf III: Proceedings of the 1998 World Scientific Congress of Golf*, M.R. Farrally and A.J. Cochran, Editors, Human Kinetics, Champaign, IL, USA, pp. 407-414

Statz, R.J., 1990, "'Surlyn' ionomers for golf ball covers," *Science and Golf: Proceedings of the First World Scientific Congress of Golf*, ed. A.J. Cochran, E. & F.N. Spon, London, UK, pp. 205-212

Statz, R.J. and J.F. Hagman, 1999, "Methods for Developing New Polymers for Golf Ball Covers," *Science and Golf III: Proceedings of the 1998 World Scientific Congress of Golf*, M.R. Farrally and A.J. Cochran, Editors, Human Kinetics, Champaign, IL, USA, pp. 481-485

Tavares, G., M. Sullivan, and D. Nesbitt, 1999, "Use of Finite Element Analysis in Design of Multilayer Golf Balls," *Science and Golf III: Proceedings of the 1998 World Scientific Congress of Golf*, M.R. Farrally and A.J. Cochran, Editors, Human Kinetics, Champaign, IL, USA, pp. 473-480