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A Virtual Golf Robot for Golf Equipment Simulation

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Abstract

The equations of motion for a two-lever pendulum are developed using Lagrange's equation. An assumed kinematic golf swing is used to generate generalized forces to drive the golf robot. These moments are used to generate a golf robot swing using LS-DYNA. The LS-DYNA model is flexible enough so that the model can be used as a virtual laboratory.

1 Introduction

In the golf industry, equipment is often evaluated using a robot that swings the club to hit the ball. The well known and established robot, called the Iron Byron, is a machine that can be modelled as a two-link swing. That is, the left arm is pinned at the center of the swing arc and connects to the club with another pin joint. The robot starts stationary from the top of the swing and applied moments at the pins drive the rotation the club. Even though it is well known that the Iron Byron (two-lever model) does not closely emulate the human golf swing, it is still widely used.

One of the most simplistic ways to model a golf swing is with two levers. The first lever represents a right-handed golfer's left arm and is pinned at the shoulder end. At the end representing the hands, a pin joint connects the left arm to the club. Early and recent work using this model involves rigid body levers [1, 2, 3]. The two lever model has two major deficiencies: the golf club is far from rigid, and only test robots, like the Iron Byron, swing this way. Advantages of the two lever model of the golf swing with rigid elements are in the relatively simplistic solutions that may be obtained using Lagrangean dynamics (or Newtonian as well). Closed form solutions for the equations of motion may be obtained for over simplistic generalized moment functions applied at the shoulder and wrist [1, 2]. Because of this, numerical integration of the equations of motion is required for a realistic solution.

Attempts have been made to determine functional form of the generalized moments applied at the shoulder and wrist. This has been done by using stroboscopic photography to determine the path of the hands and club head as a function of time. Using this kinematic data, the forces were calculated from the equations of motion.

To address the problem of a rigid club not being realistic, Winfield and Soriano have have used finite elements to represent the shaft in what was essentially a two lever model [4]. This model was driven by a kinematic assumption on the two generalized degrees of freedom. In addition to having a flexible shaft in this model, the club lever had added fidelity in that the shaft mass was distributed and the club head was a point mass.

In this paper the Lagrangean equations of motion are developed for a two lever model. A kinematic swing is considered in which the arm and club angles are given prescribed generalized displacements and the required generalized forces computed. This insures the proper release of the golf club and meeting of the ball. Inertial properties of the robot and club are used to compute moments to drive the robot. These moments are applied by using load curves and LOAD_RIGID_BODY commands.

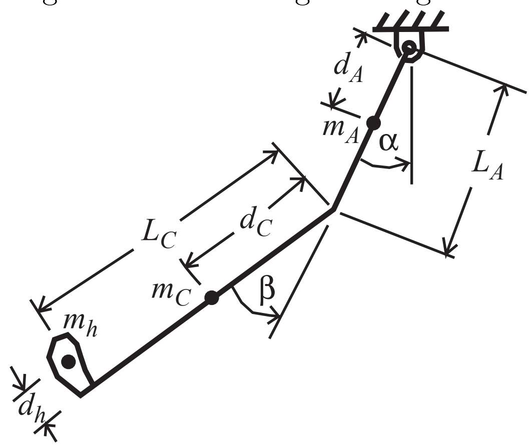
2 Two Pendulum Model

Consider the two lever model as two pendulums pinned together to model the robot golf swing. The upper link, which represents the arm, is pinned at the upper end (shoulder) which is considered the center of the swing. Label this body A and denote its length as L_A. Define the distance from the pinned end to the center of gravity of body A to be d_A. The angular position of body A is defined by the angle α it makes with the downward vertical direction. Furthermore, let the arm have mass m_A and moment of inertia about the pinned end I_A. Note that the moment of inertia about the arm's center of gravity is given by $\bar{I}_A = I_A - m_A d_A^2$.

The grip end of the golf club is connected to the arm, body A, with a pinned connection. Label the club as body C and denote its length as L_C. The distance from the grip end of the club to the club's center of gravity is defined as d_C. Let the angle β define the angular position of the club and be defined as the angle the club makes with the downward vertical direction. The mass of the club is m_C and the moment of inertia about the club's center of gravity is $\bar{I}_C$.

Having defined the two lever golf swing model as above, the equations of motion can be derived and solved subject to various initial and loading conditions. Lagrange's equations provide a convenient means for deriving the basic differential equations and are based on

Figure 1: Two-lever golf swing model



the Lagrange function defined by

$$L = T - V \tag{1}$$

where T is the kinetic energy and V is the potential energy. The equation of motion for the i^{th} generalized coordinate, q_i is given symbolically by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{2}$$

where Q_i is the generalized force corresponding to the coordinate q_i. In the two lever golf swing model $q_i = \{\alpha, \beta\}$ with corresponding shoulder and wrist moments, Q_α and Q_β, as their generalized forces.

The kinetic energy for the two lever swing model described above is

$$T = \frac{1}{2} I_A \dot{\alpha}^2 + \frac{1}{2} m_C \bar{v}_C^2 + \frac{1}{2} \bar{I}_C \left(\dot{\alpha}^2 + \dot{\beta}^2\right) \tag{3}$$

where $\bar{v}_C$ is the velocity of the club's center of gravity. The velocity component $\bar{v}_c$ is easily eliminated from the kinetic energy equation using the two lever geometry definitions. Thus, the kinetic energy may be written in terms of angles α and β and their derivatives as

$$T = \tfrac{1}{2} I_A \dot{\alpha}^2 + \tfrac{1}{2} m_C \left[L_A^2 \dot{\alpha}^2 + d_C^2 \left(\dot{\alpha} + \dot{\beta} \right)^2 + 2 L_A d_C \dot{\alpha} \left(\dot{\alpha} + \dot{\beta} \right) \cos\beta) \right] \\ + \tfrac{1}{2} \bar{I}_C \left(\dot{\alpha}^2 + \dot{\beta}^2 \right) \tag{4}$$

where $\theta = \beta - \alpha$. It is convenient to write the kinetic energy in the form

$$T = \mathcal{W}\dot{\alpha}^2 + \mathcal{X}\dot{\alpha}\dot{\beta} + \mathcal{Y}\dot{\beta}^2 + \mathcal{Z}\left(\dot{\alpha}^2 + \dot{\alpha}\dot{\beta}\right)\cos\beta \tag{5}$$

where

$$\begin{aligned} \mathcal{W} &= \tfrac{1}{2} I_A + \tfrac{1}{2} m_C L_A^2 + \tfrac{1}{2} \bar{I}_C + \tfrac{1}{2} m_C d_c^2 \\ \mathcal{X} &= \bar{I}_C + m_C d_C^2 \\ \mathcal{Y} &= \tfrac{1}{2} \bar{I}_C + \tfrac{1}{2} m_C d_C^2 \\ \mathcal{Z} &= m_C L_A d_c \end{aligned} \tag{6}$$

The potential energy for the two lever swing model is the same as the potential energy for a simple double pendulum whose plane is tilted at approximately $\eta = 57^o$ with the vertical to represent the swing plane. Thus,

$$V = \left[m_A g d_A \left(1 - \cos\alpha\right) + m_C g L_A \left(1 - \cos\alpha\right) \\ + m_C g d_C \left(1 - \cos\beta\right) \right] \sin\eta \tag{7}$$

where g is the acceleration due to gravity.

It has been shown that the the force on the club due to gravity does not greatly influence the swing [3]. Thus, the potential energy terms are neglected in the modelling of this paper. Hereafter, the Lagrangian function L is replaced the the kinetic energy T for deriving the equations of motion.

Differentiation of the kinetic energy (Eq. 5) as prescribed by Lagrange's equation (Eq. 2) leads to the following equations of motion:

$$2\mathcal{W}\ddot{\alpha} + \mathcal{X}\ddot{\beta} + \mathcal{Z}\left(2\ddot{\alpha} + \ddot{\beta}\right)\cos\beta = Q_\alpha \tag{8}$$

$$\mathcal{X}\ddot{\alpha} + 2\mathcal{Y}\ddot{\beta} + \mathcal{Z}\ddot{\alpha}\cos\beta + \mathcal{Z}\left(\dot{\alpha}^2 + \dot{\alpha}\dot{\beta}\right)\sin\beta = Q_\beta \tag{9}$$

A final generalized force required to implement the golfer's wrist roll may be written in terms of angle γ measuring the location of the club face. The equation of motion for this angle is

$$\left(\bar{I}_{Cr} + m_h d_h^2\right)\ddot{\gamma} = Q_\gamma \tag{10}$$

where I_{Cr} is the moment of inertia of the shaft about its long axis, m_h is the mass of the club head and d_h is the distance of the club head's center of mass to the club shaft axis.

Generalized forces Q_α, Q_β, and Q_γ will be the applied moments used to drive the LS-DYNA robot model.

3 Kinematic Driven Swing

A simple way to swing the golf club is to prescribe the angles α and β as a function of time. In general, the club is at rest at the top of the swing and accelerates in the downswing until reaching a maximum at the ball. Taking this acceleration to be gradual is a good assumption for a proper golf swing. Thus, a good starting point for modelling the downswing a sinusoidal function.

Let the total time of the downswing be designated by t_a. A second time critical for the swing of the golf club is the time at which the wrists unhinge, t_r. In time t_a the golfer's arm goes from the top-of-the-swing position to the impact position. This angle will depend on the degree the golfer takes the club back. For a swing having the club parallel to the ground at the top of the swing the angle α will go through $\pi/2$ radians. (This is based on a wrist cock angle of $\beta_o = \frac{\pi}{2}$ which is subsequently defined.) Any amount the club is short of parallel is defined as α_o. The above can be accomplished by assuming the following definition for α

$$\alpha(t) = A\left(t - \frac{t_a}{\pi} sin\frac{\pi}{t_a}t\right) + \alpha_o - \frac{\pi}{2} \tag{11}$$

with $t = 0$ representing the top of the swing and $t = t_a$ representing impact. Observe that $\alpha(0) = \alpha_o - \pi/2$ and $\alpha(t_a) = 0$ as desired if

$$A = \frac{\pi}{2t_a} - \frac{\alpha_o}{t_a} \tag{12}$$

To model the kinematics of the wrist unhinging, define time duration t_b as the time period between unhinging and impact with the ball

$$t_b = t_a - t_r \tag{13}$$

Before the wrists unhinge the angle β is fixed and simply defined as

$$\beta = \beta_o = -\frac{\pi}{2} \qquad \forall \;\; t < t_r \tag{14}$$

It should be noted that a range of angles can be taken as the wrist cock angle, β_o, rather than just $\frac{\pi}{2}$. As the wrists unhinge the angle β is written as

$$\beta(t') = B\left(t' - \frac{t_b}{\pi} sin \frac{\pi}{t_b} t'\right) - \beta_o \tag{15}$$

where $t' = t - t_r$ and $B = \frac{\beta_o}{t_b}$.

Note that both A and B have units of rad/s, that is, angular velocity units. Differentiation of α and β results in impact angular velocities of $2A$ and $2B$ for $\dot{\alpha}$ and $\dot{\beta}$, respectively.

The wrist roll also occurs after the wrist release. Thus,

$$\gamma(t') = C\left(t' - \frac{t_b}{\pi} \sin \frac{\pi}{t_b} t'\right) - \frac{\pi}{2} \tag{16}$$

where $t' = t - t_r$ and $C = \frac{\pi}{2t_b}$.

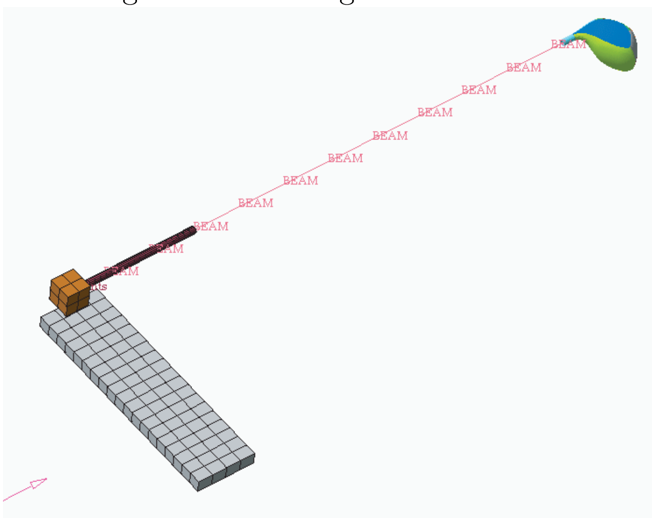
Using Eqs. 11, 15, and 16 in Eqs. 8 - 10 gives a way to compute moments Q_α, Q_β, and Q_γ.

4 LS-DYNA Model of the Two-lever Swing

The model of the two-lever golf swing was based on a hitting robot used in club testing. In this machine, the arm portion of the model was taken as a 25.4 mm thick rigid block of solid elements which is 482.6 mm long by 101.6 mm wide. This part rotates about the middle of one end that is the center of rotation for the two-lever swing. Nodes on top and bottom of the plate are pinned using CONSTRAINED_NODE allowing for fixed-axis rotation. The arm is driven by an applied moment, Q_α, to the arm using the LOAD_RIGID_BODY command.

At the end of the arm opposite to the rotation center is a rigid block of solid elements used to implement the wrist release of the golf swing. This block is a 50.8 mm cube whose dimension was taken to approximate the corresponding part of an Iron Byron robot. The wrist release block is attached to the arm using a constrained revolute joint. Since the wrist release has to maintain $\beta = \beta_o$ during the initial

Figure 2: Overall golf robot model



part of the downswing a contact constraint was used. Segments were defined on both the arm and wrist release block and tied together. This contact definition is given a death time corresponding to the wrist release time t_r. The wrist roll, like the arm, was driven by an applied moment, Q_β, to the rigid body (LOAD_RIGID_BODY).

The final piece of the robot is the collet that holds the golf club. This part was modelled using rigid shell elements in a cylindrical shape. The collet attaches to the wrist release block using a constrained revolute joint. In addition to attaching the club to the robot, the collet provides the wrist roll that squares the club face to the ball (hence, the revolute joint). To keep the club from twisting during the downswing, a segment contact was defined tieing the wrist roll until t_r. Squaring the club face after the wrist release is accomplished by applying a moment, Q_γ via a LOAD_RIGID_BODY. For the wrist roll a follower moment was used.

A club is placed in the robot by constraining one or more shaft nodes to the collet (CONSTRAINED_EXTRA_NODES). When finding the moment of inertia, mass center location, and mass for the above analysis, the inertia properties of the collet must be added to those of the club. Figure 2 shows the overall model (with the exception of the ball) while Figure 3 shows a detail of the arm-wrist-collet portion including revolute joints and tied contact definitions.

In the above model, moments Q_α, Q_β, and Q_γ are input using load curves. These curves can be generated using Eqs. 8, 9, and 10. The inertia properties of the robot and the club was determined from running the model for a short period and then examining the D3HSP file. It is noted that the inertia properties of the club as defined above needs to include those of wrist block, collet, shaft, and club head. For the wrist roll, the inertia term consists of the collet, shaft, and club head.

It should be noted that moments Q_β and Q_γ generated from Lagrange's equations are nonzero prior to the wrist release. Since there are contact definitions active in this time period, the load curves are taken as zero before the wrist release. Immediately after the wrist release the load curve is taken to be value computed by Lagrange's equation.

Figure 3 shows input parameters for a swing along with the form of the arm and wrist moments. Using the load curves generated for Q_α, Q_β, and Q_γ a LS-DYNA simulation shows the effectiveness of model.

5 Conclusion

The equations of motion for a double pendulum were derived to model a golf robot swing. Generalized moments from the resulting equations were computed and used to drive an LS-DYNA model of the robot. This provides an effective way to generate many different swing profiles for swinging the virtual golf robot.

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Figure 3: Detail of revolute joints and tied contact definitions

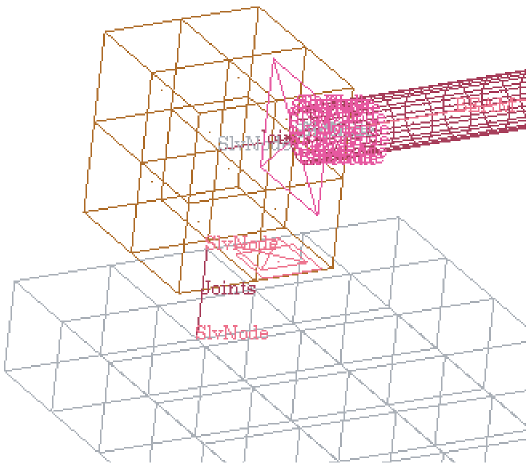

Figure 4: Panel used to input parameters and generate moment curves